# BIZness: Bit Invariant Zero-Sum Property based on Division Trail

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# DiFA [KGSR23] The Source

- Bit invariance $\implies$ same output division property appears regardless of the position of active bits in the input division property.

## Results (5 rounds)

- For PRESENT, BIZ set: $\{0, 4, 8, 12\}$.
- For GIFT-64, BIZ set: $\{0, 4, 8, \cdots, 60\}$.
- For GIFT-128, $\exists$ 2 distinct sets. $|\text{BIZ}| = 80$.

| Primitive | round | # sets | #BIZ |
|---|---|---|---|
| PRESENT-80/128 | 4 | 1 | 64 |
| | 5 | 1 | 4 |
| GIFT-64 | 4 | 1 | 64 |
| | 5 | | 16 |
| GIFT-128 | 4 | 1 | 128 |
| | 5 | 2 | 80 |

# Background

## Definition (Bit-Based Division Property [Tod15])

A multi-set $X \subseteq \mathbb{F}_2^n$ is said to have the bit-based division property $\mathcal{D}_{\mathbb{K}}^n$ for some set of $n$-dimensional vectors $\mathbb{K}$, if it fulfills the following conditions:

$$\bigoplus_{\boldsymbol{x} \in X} \boldsymbol{x}^{\boldsymbol{u}} = \begin{cases} \text{unknown} & : \text{if there is } \boldsymbol{k} \in \mathbb{K} \text{ s.t } \boldsymbol{u} \succeq \boldsymbol{k} \\ 0 & : \text{otherwise} \end{cases}$$

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## Definition (Balanced Bit [Tod15])

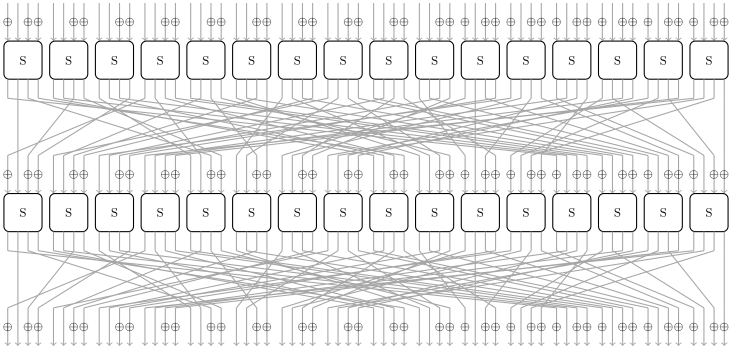
Let $Y \subseteq \mathbb{F}_2^n$ be a multi-set of vectors. A bit position $0 \leq i < n$ is called balanced bit position if $\bigoplus_{y \in Y} y_i = 0$

## Zero-sum property

If for any boolean function $f : \mathbb{F}_2^n \rightarrow \mathbb{F}_2^n$, $\exists\ I \subseteq \mathbb{F}_2^n$ s.t.

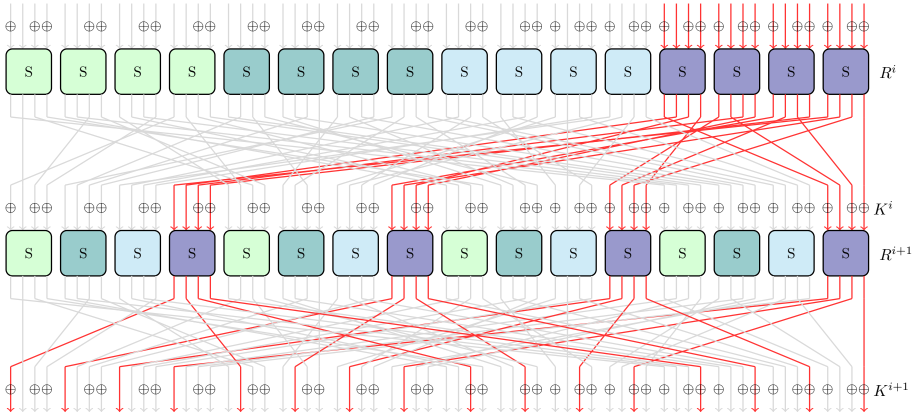
$$\bigoplus_{x \in I} x = \bigoplus_{x \in I} f(x) = 0$$

# GIFT [BPP+17] Recall




- Two versions GIFT-64 and GIFT-128 with state size 64 and 128 bits respectively.
- Consists of 28 and 40 rounds respectively.
- SubCells, Bit Permutation, Addition of round keys in each of the round function.
- SBox has branch number 2.
- Introduces the BOGI permutation (bad outputs goes to good inputs).

# QR-Structure of GIFT Permutation




- Property of two consequitive rounds.
- Permutation within 16-bits.
- Maps the output bits of SBoxes from a quotient group to the input bits of SBoxes in the corresponding remainder group.

# Our Contribution

- We develop a framework for appearing bit invariance zerosum property.
- The property manifests due to both the Sbox and the permutation layer.
- We give the explanation using Algebraic Normal Form (ANF).
- The charecterization of BIZ is done by combining different SBoxes with GIFT and PRESENT permutation layer.
- The property remains unchanged after replacing the GIFT BOGI permutaion with any 16-bit BOGI permutation.

# Division Trail Induced Linear Structures - DiviLS

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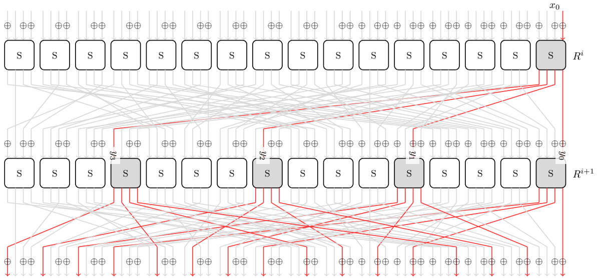
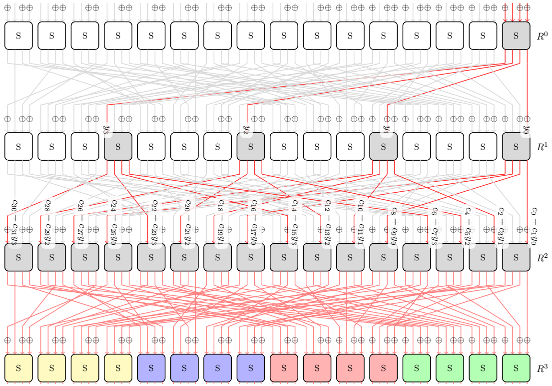
## Observations

- **Sbox-DiviLS:** One input bit of an SBox has DiviLS $\implies$ every SBox output exhibit a DiviLS.
- **QR-DiviLS:** One SBox of a quotient group has a DiviLS $\implies$ each of the four SBoxes in corresponding remainder group has a DiviLS.



# 2.5-Round DiviLS in GIFT

- For the input set corresponding to a division property limited to a nibble at $R^i$, the DiviLS property holds up to $R^{i+2}$.




- This holds for bit permutation layer of PRESENT also.

# BIZ in GIFT-64

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- BIZ = Bit Invariance Zerosum

**Property [KGSR23]**

Irrespective of the position of the active nibble in the input division set of GIFT-64, after 5 rounds, ∃ specific 16 bits, one in every nibble, that are balanced.

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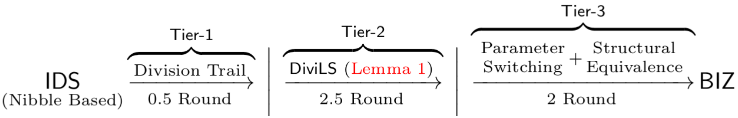
- BIZ = Bit Invariance Zerosum

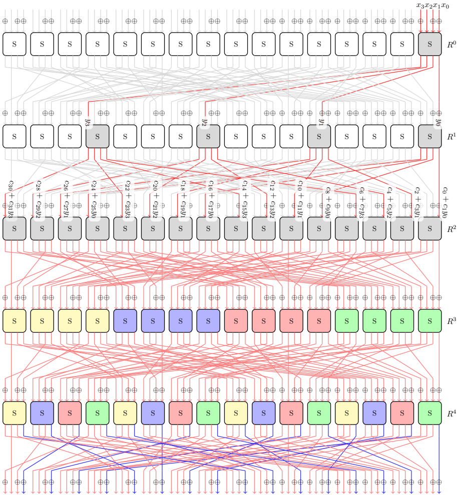
**Property [KGSR23]**

Irrespective of the position of the active nibble in the input division set of GIFT-64, after 5 rounds, $\exists$ specific 16 bits, one in every nibble, that are balanced.

- Try to give a theoretical argument behind this Property.
- Use a three-tier approach:



# BIZ in GIFT-64




- **Tier-2:**
  - Suppose $\{x_3^0, x_2^0, x_1^0, x_0^0\}$ := input bit variables at $R^0$.
  - Consider the variables $\nu = \{y_3^0, y_2^0, y_1^0, y_0^0\}$, after the permutation layer of $R_0$.
  - At input of $R^3$, variables are linear over $\nu$ (2.5 round DiviLS of GIFT-64).
  - Any SBox before $R^3$ can have degree at most 3.

# BIZ in GIFT-64

- **Parameter Switching ($\mathcal{P}_f(\mathbf{p} \to \mathbf{q})$):**
  - Change the parameters $\mathbf{p} \to \mathbf{q}$ in a boolean funtion $f(\mathbf{p}, \mathbf{x})$ by $p_i$ with $q_i$, s.t. $\mathbf{p} \neq \mathbf{q}$ in the ANF of $f$.
  - **Example:** Consider,

$$f(c_2, c_1, c_0, d_2, d_1, d_0, x_3, x_2, x_1, x_0) = d_0x_0 + d_2d_1x_2x_1 + c_2d_1x_1 + c_1c_0d_0x_3,$$

with $\mathbf{c}$, $\mathbf{d}$ are parameters.
Take, $g = \mathcal{P}_f(\mathbf{c} \to \mathbf{a}, \mathbf{d} \to \mathbf{b})$, then,

$$g(a_2, a_1, a_0, b_2, b_1, b_0, x_3, x_2, x_1, x_0) = b_0x_0 + b_2b_1x_2x_1 + a_2b_1x_1 + a_1a_0b_0x_3.$$

# BIZ in GIFT-64

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  - Change the parameters **p** → **q** in a boolean funtion $f(\mathbf{p}, \mathbf{x})$ by $p_i$ with $q_i$, s.t. **p** ≠ **q** in the ANF of $f$.
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- **Structural Equivalence ($f \equiv g$):**
  - For parameterized Boolean functions $f(\mathbf{p}, \mathbf{x})$ and $g(\mathbf{q}, \mathbf{x})$,

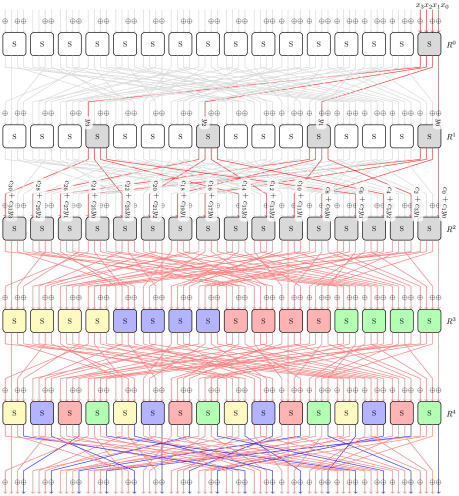
$$\text{structurally equivalent} \iff \mathcal{P}_f(\mathbf{p} \to \mathbf{q}) = g.$$

  - **Example:** Consider the following functions:

$$f(c_1, c_0, x_3, x_2, x_1, x_0) = x_0 + x_2x_1 + c_0x_1 + c_1x_2$$
$$g(d_1, d_0, x_3, x_2, x_1, x_0) = x_0 + x_2x_1 + d_0x_1 + d_1x_2.$$

    Here, $\mathcal{P}_f(\mathbf{c} \to \mathbf{d}) = g(\mathbf{d}, \mathbf{x})$ and so $f \equiv g$.

# BIZ in GIFT-64

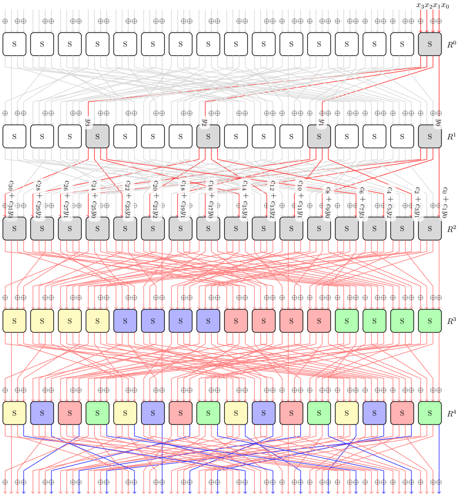



- **Tier-3:**
  - From Tier-2, before $R^3$, all the inputs to the SBoxes in a specific quotient group becomes structurally equivalent.
  - The outputs of four SBoxes in each quotient group are also structurally equivalent to each other.

$$\begin{cases} y^3_{4(4j+0)+0} \equiv y^3_{4(4j+1)+0} \equiv y^3_{4(4j+2)+0} \equiv y^3_{4(4j+3)+0} \\ y^3_{4(4j+0)+1} \equiv y^3_{4(4j+1)+1} \equiv y^3_{4(4j+2)+1} \equiv y^3_{4(4j+3)+1} \\ y^3_{4(4j+0)+2} \equiv y^3_{4(4j+1)+2} \equiv y^3_{4(4j+2)+2} \equiv y^3_{4(4j+3)+2} \\ y^3_{4(4j+0)+3} \equiv y^3_{4(4j+1)+3} \equiv y^3_{4(4j+2)+3} \equiv y^3_{4(4j+3)+3} \end{cases}$$

  - Apply a parameter switching to get structurally equivalent inputs to the SBoxes of $R^4$.

# BIZ in GIFT-64




- **Tier-3:**
  - Consider $\mathbf{u} = (u_0, u_1, u_2, u_3)$, the four output bits of any SBox in $R^4$ i.e. Then,

$$(a_0^i y_0 + a_1^i, b_0^i y_0 + b_1^i,$$
$$c_0^i y_0 + c_1^i, \ \ d_0^i y_0 + d_1^i)$$
$$\downarrow (SBox \circ \mathcal{F} \circ SBox)$$
$$(u_0, u_1, u_2, u_3)$$

where,

$$\mathcal{F} = \begin{cases} \mathcal{P}_{y^3_{4(4j+0)+0}}(\mathbf{a}^i \to \mathbf{a}^i, \mathbf{b}^i \to \mathbf{b}^i, \mathbf{c}^i \to \mathbf{c}^i, \mathbf{d}^i \to \mathbf{d}^i) \\ \mathcal{P}_{y^3_{4(4j+1)+1}}(\mathbf{a}^i \to \mathbf{a}^{i+1}, \mathbf{b}^i \to \mathbf{b}^{i+1}, \mathbf{c}^i \to \mathbf{c}^{i+1}, \mathbf{d}^i \to \mathbf{d}^{i+1}) \\ \mathcal{P}_{y^3_{4(4j+2)+2}}(\mathbf{a}^i \to \mathbf{a}^{i+2}, \mathbf{b}^i \to \mathbf{b}^{i+2}, \mathbf{c}^i \to \mathbf{c}^{i+2}, \mathbf{d}^i \to \mathbf{d}^{i+2}) \\ \mathcal{P}_{y^3_{4(4j+3)+3}}(\mathbf{a}^i \to \mathbf{a}^{i+3}, \mathbf{b}^i \to \mathbf{b}^{i+3}, \mathbf{c}^i \to \mathbf{c}^{i+3}, \mathbf{d}^i \to \mathbf{d}^{i+3}) \end{cases}$$

# BIZ in GIFT-64

- **Tier-1:**
  - Til now, we have considered the propagation of the variables $y_i^0$ for $i \in \{0, \cdots, 3\}$.
  - Actual input variable $:= (x_3, x_2, x_1, x_0)$ before $R^0$.
  - Use the division property table to check if any of the output bits contain the monomial $x_3x_2x_1x_0$ or not.

**Lemma**

If from the input property 1111, $\exists$ only 1111 in the division property table of the SBox and the $i$-th output bit does not contain the monomial $y_3y_2y_1y_0$, then after 5 rounds the $i$-th output bit of GIFT is balanced.

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- Algorithm for checking the conditions:

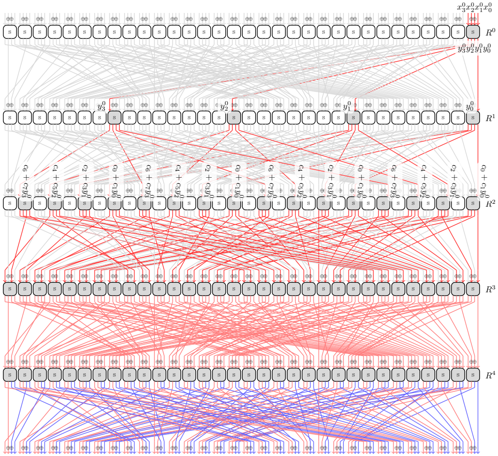
**Algorithm 1** CHECK BIZ FOR GIFT-64

Input: An SBox $\mathcal{S}$, the division property table $DPT_{\mathcal{S}}$ where $(b, a) \in DPT_{\mathcal{S}}$ $\iff$ $(b, a)$ is a valid division trail for $\mathcal{S}$.
Output: coordinates with BIZ property

1: $(u_0, u_1, u_2, u_3) = SBox \circ \mathcal{F} \circ SBox(a_0^i y_0 + a_1^i, b_0^i y_0 + b_1^i, c_0^i y_0 + c_1^i, d_0^i y_0 + d_1^i)$
2: $B = \phi$
3: **if** $(1111, a) \notin DPT_{\mathcal{S}}$ such that $a \neq 1111$ **then**
4:     **for** $i = 0$ to $4$ **do**
5:         **if** $deg(u_i) < 4$ **then**
6:             $B = B \cup \{i\}$
7: **return** B

# BIZ in GIFT-128 and PRESENT

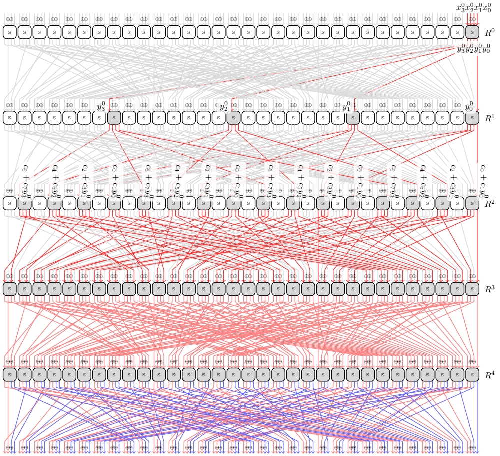
# BIZ in GIFT-128




## Result [KGSR23]

- The two sets of bit invariance balanced bits for GIFT-128.
- Two set appears depending upon the initial active nibble, say $\mathcal{A}$ and $\mathcal{B}$.
- $|\mathcal{A} \cap \mathcal{B}| = 64$

# BIZ in GIFT-128

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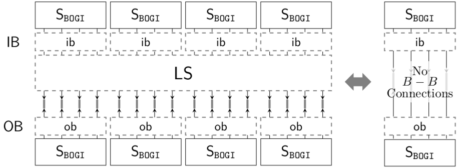
- $\mathcal{A}$, $\mathcal{B}$ and BIZ set can be obtained by previously discussed similar procedure.
- For PRESENT also, the same technique can be used to show the BIZ property for 5 rounds of the cipher.

# BIZ in other Permutation Layer [KHSH22]

- Proposed a clasification of BOGI-based ciphers.
- Decomposed, BOGI $\equiv OB \circ LS \circ IB$, where $IB$ and $OB$ are 4-bit permutations and $LS$ is 16-bit permutation.




- For BOGI permutation, $LS$ satisfy the following conditions:
  - The four input bits of LS in each SBox position go to four distinct SBox positions.
  - Each bit order of the four input bits of LS in each SBox position is invariant on the four output bits of LS in an SBox position.
- Total BOGI-based cipher can be categorized into 24 classes.

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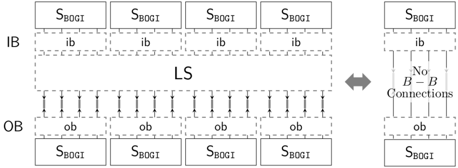


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## Our Result

- We replace the bit permutation of GIFT-64 with each of the 24 permutations.
- The BIZ property remains invariant and appears in the exact same bit positions for each permutation after 5 rounds.

# BIZ after varying SBox

| SBox | PRESENT Permutation | | | GIFT-64 Permutation | | | GIFT-128 Permutation | | |
|---|---|---|---|---|---|---|---|---|---|
| | Round | #bb | BIZ | Round | #bb | BIZ | Round | #bb | BIZ |
| Craft | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 16 | ✓ | 5 | 0 | ✗ | 5 | 32 | ✗ |
| Midori-128 | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 0 | ✗ | 5 | 0 | ✗ | 5 | 0 | ✗ |
| Prince | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 0 | ✗ | 5 | 0 | ✗ | 5 | 0 | ✗ |
| Rectangle | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 16 | ✓ | 5 | 16 | ✓ | 5 | 80 | ✓ |
| Small AES | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 0 | ✗ | 5 | 0 | ✗ | 5 | 0 | ✗ |
| PRESENT | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 4 | ✓ | 5 | 0 | ✗ | 5 | 0 | ✗ |
| GIFT | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 16 | ✓ | 5 | 16 | ✓ | 5 | 80 | ✓ |
| Piccolo | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 32 | ✓ | 5 | 32 | ✓ | 5 | 64 | ✓ |
| Warp | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 16 | ✓ | 5 | 0 | ✗ | 5 | 32 | ✗ |
| Default | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 48 | ✓ | 5 | 32 | ✓ | 5 | 128 | ✓ |
| Baksheesh | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 4 | 64 | ✓ | 5 | 64 | ✓ | 5 | 128 | ✓ |
| ULBC | 4 | 64 | ✓ | 4 | 64 | ✓ | 4 | 128 | ✓ |
| | 5 | 24 | ✓ | 5 | 16 | ✓ | 5 | 64 | ✓ |

# Conclusion

- We know the reason for appearing the BIZ property.
- The designers should consider this before proposing a new GIFT-like structure.


thank you
danke
gracias
merci
obrigado
bedankt
dziękuję
teşekkür ederim
go raibh maith agat
ngiyabonga
謝謝
спасибо
sukriya
kop khun krap
terima kasih
grazie
arigatô
dakujem
mochchakkeram
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nanni
nandri
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